MPM2D
Examination
Exam A, 2012
Length: 3 hours
(Exam set for 2 hrs. + 1 hr. flex time)

OTTAWA-CARLETON
DISTRICT SCHOOL BOARD

Name : Answer Sheet

Teacher : ____________________

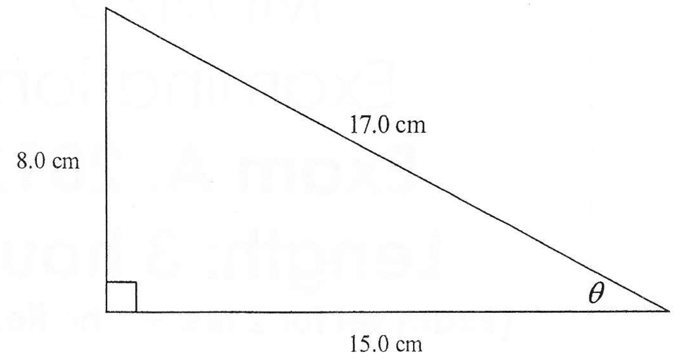
School : ____________________

Instructions to students:

1. This examination booklet is **13 pages** long.
   Please check that you have all the pages.
2. Answer all questions with complete solutions in the spaces provided on the examination paper.
3. You may use any school-approved calculator on this examination.
   Make sure that your calculator is in **DEGREE** mode.
   Do **<u>not</u>** share your calculator.
4. There is a formula sheet that goes with the examination.
5. Diagrams are not drawn to scale.

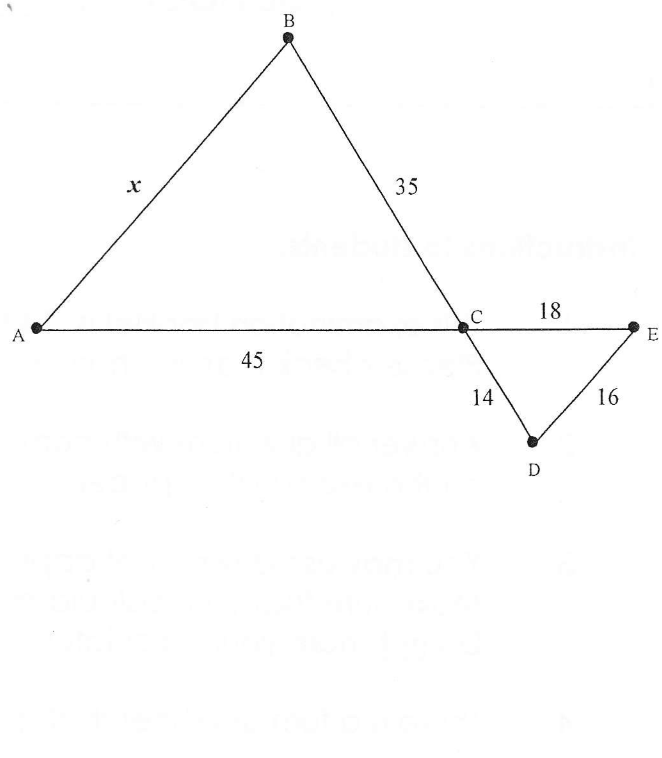
A) Trigonometry

A1) **Determine** the value of θ to the nearest degree.



$$\theta = \tan^{-1}\left(\frac{8}{15}\right) = \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{15}{17}\right) \doteq 28°$$

A2) **Determine** the value of x.



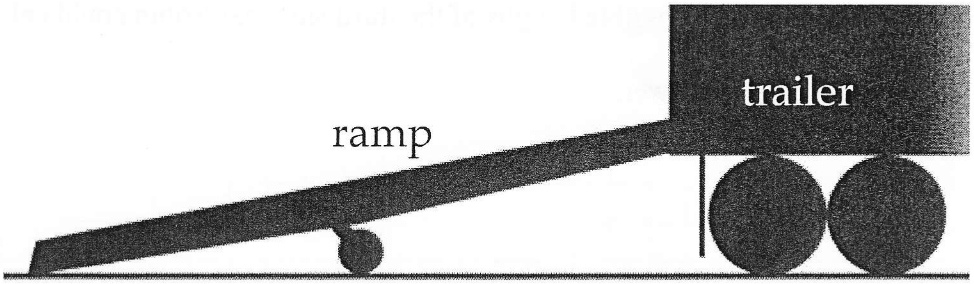
$$\frac{AC}{EC} = \frac{BC}{DC} = \frac{5}{2} \text{ and } \angle ACB = \angle ECD \text{ by OAT}$$

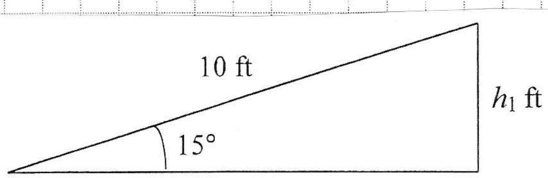
$$\therefore \Delta ABC \sim \Delta EDC \text{ by SAS}\sim$$

$$\frac{x}{16} = \frac{5}{2}$$

$$x = 40$$

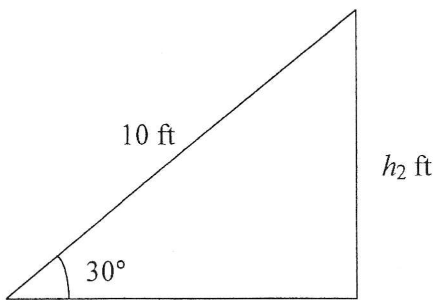
3) Rick can place a 10 foot long ramp at an angle of 15° to reach a trailer.
Roy thinks that to reach a trailer twice as high as the first, the angle should be 30°.
Do you agree with Roy?
Justify your answer.





$$\frac{h_1}{10} = \sin 15°$$

$$h_1 \doteq 2.6$$



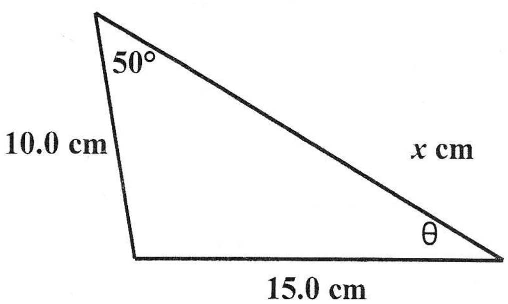
$$\frac{h_2}{10} = \sin 30°$$

$$h_2 = 5$$

Even though the angle has doubled, the height is not doubled.
Sine ratios are not linearly related to the angle.

A4) Robin forgot her homework again so she calls Scott and he tells her about it.
She has to determine the length of the third side of a non-right triangle if one of the angles is 50° and other sides are 15.0 cm and 10.0 cm
Scott forgot to tell her which angle was 50°.
Determine <u>all possible lengths</u> of the third side that Robin could calculate, rounded to one decimal place.
Justify your answer.

Three possibilities:

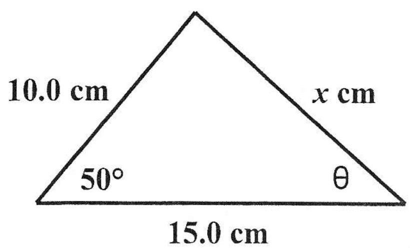


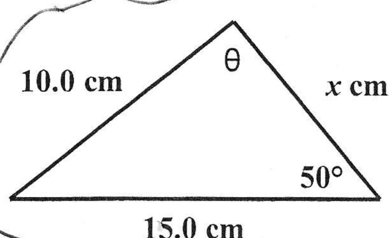
Need to find the angle opposite the 10.0 cm side, θ.

$$\theta = \sin^{-1}\left(\frac{10\sin 50^{\circ}}{15}\right) \doteq 30.7^{\circ}$$

$$x = \frac{15\sin(180^{\circ} - 50^{\circ} - \theta)}{\sin 50^{\circ}}$$
$$\doteq 19.3$$

$$x = \sqrt{10^2 + 15^2 - 2(10)(15)\cos 50^{\circ}}$$
$$\doteq 11.5$$





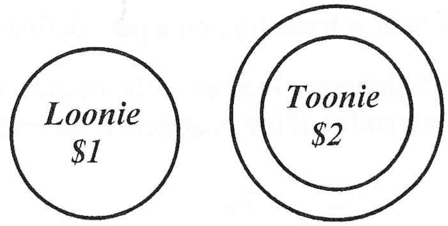
Need to find the angle opposite the 15.0 cm side, θ.

$\theta = \sin^{-1}\left(\frac{15\sin 50^{\circ}}{10}\right)$ which results in an error.

☆ There is no triangle possible in this case.

B) Analytic Geometry

B1) Phil has 83 coins made up of Loonies and Toonies.
He has a total of \$137.
Determine the number of Loonies that Phil has.



let x represent loonies.
let y represent toonies.

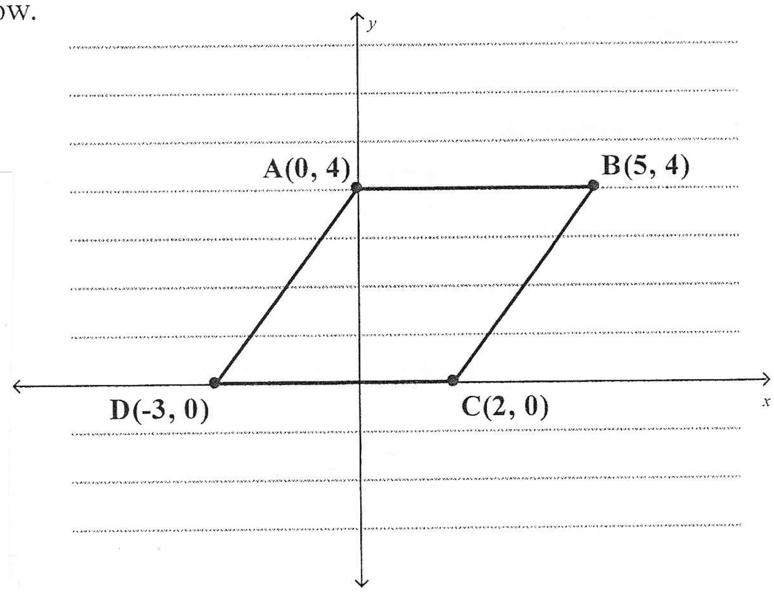
$$
\begin{aligned}
① \quad x + y &= 83 \\
- \quad ② \quad x + 2y &= 137 \\
\hline
-y &= -54 \\
\frac{-y}{-1} &= \frac{-54}{-1} \\
y &= 54
\end{aligned}
$$

sub $y = 54$ into ①

$$
\begin{aligned}
x + 54 &= 83 \\
x &= 83 - 54 \\
x &= 29
\end{aligned}
$$

∴ Phil has 29 loonies

B2) **Identify** the type of quadrilateral shown below.
Justify your answer.



$m_{AB} = m_{CD} = 0$

$m_{CA} = m_{DB} = \frac{4}{3}$

$\overline{AB} = \overline{CD} = 5$

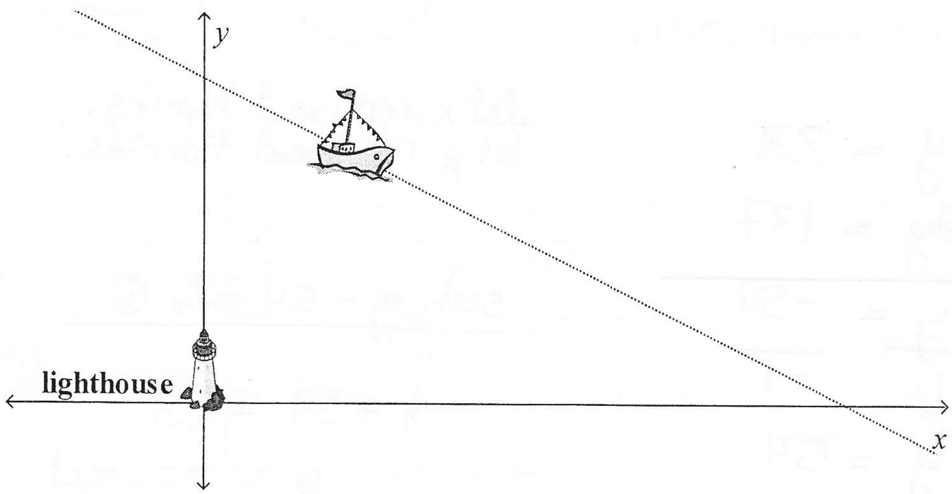
$\overline{CA} = \overline{DB} = \sqrt{3^2 + 4^2} = 5$

Quad ABCD is a rhombus since its opposite sides are parallel and adjacent sides are equal in length. It is not a rectangle or square as adjacent sides are not perpendicular.

B3) A boat is travelling on a path defined by $y = -\frac{1}{2}x + \frac{17}{2}$ where x and y are measured in kilometres.

A lighthouse, located at the origin, can detect boats up to 8 km away.

Determine if the boat gets close enough to the lighthouse to be detected.



P represents the closest point along the path of the boat to the lighthouse.

For the coordinates of P:

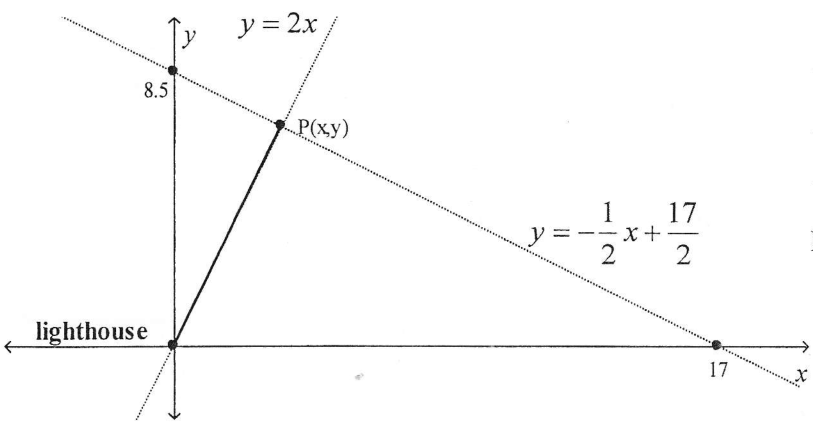
$$2x = -\frac{1}{2}x + \frac{17}{2}$$

$$\frac{5}{2}x = \frac{17}{2}$$

$$x = \frac{34}{10}$$

$$x = 3.4$$

$$y = 6.8$$



Distance from (0, 0) to P

$= \sqrt{3.4^2 + 6.8^2}$

$= \sqrt{57.8}$

$\doteq 7.6$

Since the distance is less than the range, the boat passes within the detection range of the lighthouse.

B4) For homework Barney is graphing the line with equation $5x+3y=240$.
For fun, he switched the coefficients of the variables yielding the equation $3x+5y=240$ and noticed that the two lines crossed.
Does switching the coefficients of <u>any equation</u> of this form result in one point of intersection? **Justify** your answer.

① $5x+3y=240$

$\frac{3y}{3} = \frac{-5x}{3} + \frac{240}{3}$

$\boxed{y = -\frac{5}{3}x + 80}$

$y = ax + b$

② $3x+5y=240$

$\frac{5y}{5} = \frac{-3x}{5} + \frac{240}{5}$

$\boxed{y = -\frac{3}{5}x + 48}$

$y = ax + b$

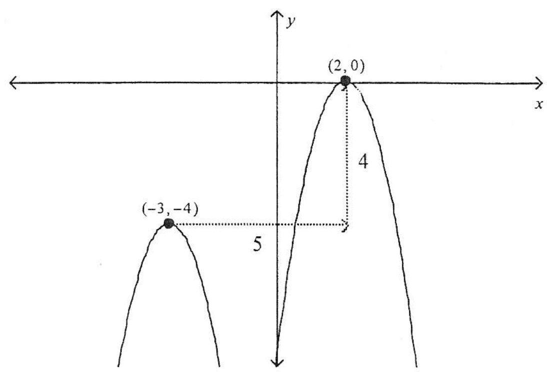
<u>Now compare</u>

If one equation has the form $ax+by=c$ then the new equation would be $bx+ay=c$.
Case 1: If $a=b$, the equations would represent lines that would be coincident and share an infinite number of points since the two lines would be identical.
Case 2: If $a=-b$, the equations would represent lines that are parallel and not the same so there would be no intersection points. The slopes of these lines would be equal but different ordered pairs would satisfy the equations.
Case 3: Any other relationship between a and b would result in equations representing lines which share exactly one point. i.e. $|a| \neq |b|$

By the way, the intersection point of the two original lines is (30, 30). In Case 3, switching the coefficients of x and y will give an intersection with equal x and y coordinates.

C) Quadratic Relations

C1) The parabola with equation $y = -2(x+3)^2 - 4$ is moved 4 units up and 5 units to the right.
State the equation of the new parabola.
Justify your answer.



$$y = -2(x-2)^2$$

C2) **Solve** the equation $10x^2 + x = 2$.

$$10x^2 + x - 2 = 0$$

$$(5x-2)(2x+1) = 0$$

$$x = -\frac{1}{2} \text{ or } x = \frac{2}{5}$$

C3) **Compare** the properties of the three quadratic relations given below.

<table>
<tr><th>Model 1</th><th>Model 2</th><th>Model 3</th></tr>
<tr><td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>–2</td><td>0</td></tr>
<tr><td>–1</td><td>5</td></tr>
<tr><td>0</td><td>8</td></tr>
<tr><td>1</td><td>9</td></tr>
<tr><td>2</td><td>8</td></tr>
<tr><td>3</td><td>5</td></tr>
<tr><td>4</td><td>0</td></tr>
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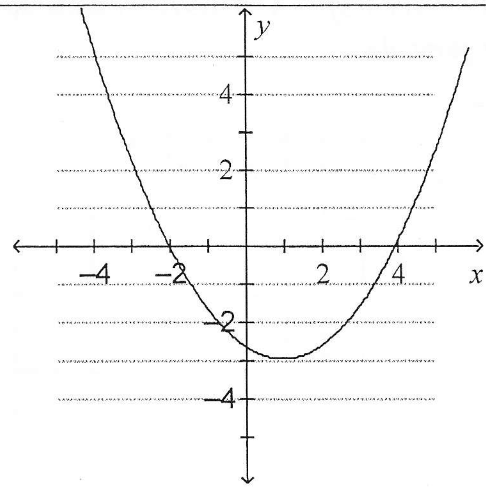
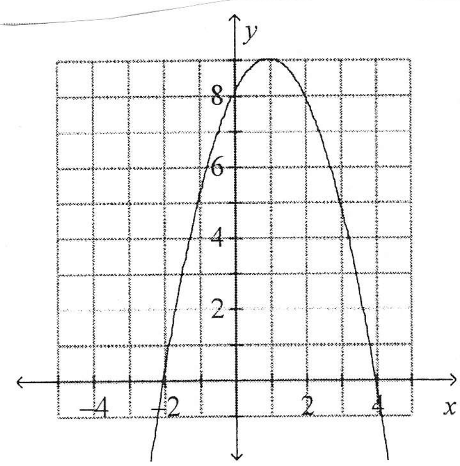
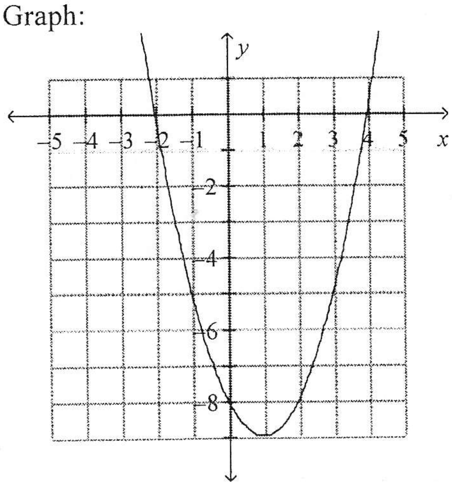
</td><td>
</td><td>$y=(x+2)(x-4)$</td></tr>
</table>



Model 2: equation $y=\frac{1}{3}(x-1)^2-3$

Model 3: equation $y=(x-1)^2-9$
Graph:

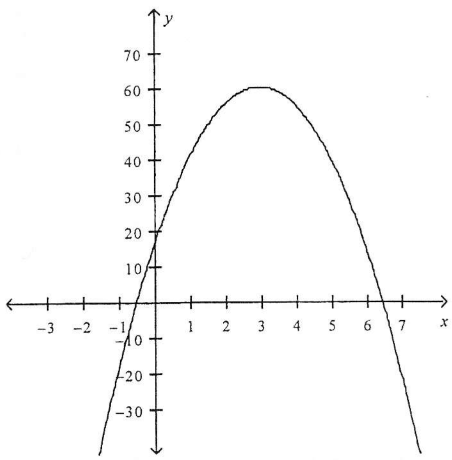


Similarities:
All three relations represent parabolas having x-intercepts at –2 and 4 and the vertex at $x=-1$. Also the axis of symmetry for each would have equation $x=-1$. Their tables of values would have equal second differences but they would not be equal to each other.

Differences:
Model 1 represents a parabola which opens down and congruent to $y=x^2$. Model 2 and Model 3 open up. The y-values of each vertex is different. Model 2 is a compression by a factor of 3 of the parent relation $y=x^2$ while Model 3 is congruent to $y=x^2$.

C4) Select values of a and h so that the quadratic relation with equation $y = a(x-h)^2 + 60$ models a real life scenario.
Explain the connection between the features of the parabola and the scenario you have modeled.

Example (answers may vary)
Let $a = -5$ and $h = 3$. The equation would be $y = -5(x-3)^2 + 60$.
This could represent the height of a golf ball in metres above ground level x seconds after the ball is hit.



The y-intercept is 15 and this would represent the height of an elevated tee 15 m above ground level.

The vertex is (3, 60) and this means that the ball reaches its maximum height of 60 m above ground level 3 seconds after it is hit.

The zeros are about -0.46 and 6.46. The negative zero is outside of appropriate time values, but the second zero represents the number of seconds after the ball is hit when it reaches ground level.